Q.10 (iv) Let $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$, Then $A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Let $P = \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

Thus, $P = \frac{1}{2}(A+A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q.$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix

$\therefore P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A.$

Q.11 If A, B are symmetric matrices of same order, then AB − BA is a

(a) skew-symmetric matrix (b) symmetric matrix
(c) zero matrix. (d) identity matrix.

Soln: It is given that A and B are symmetric matrices.

$A' = A$ and $B' = B$

$(AB - BA)' = (AB)' - (BA)'$ $[\because (A-B)' = A' - B']$

$= B'A' - A'B'$ $[\because (AB)' = B'A']$

$= BA - AB$ $[A' = A$ and $B' = B]$

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Thus, (AB-BA) is a skew-symmetric matrix. So, option (a) Ans

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Q.12. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, if the value of α is (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) π (d) $\frac{3\pi}{2}$

Soln: It is given that $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

and $A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

It is given that

$$A + A' = I$$

$$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore 2\cos\alpha = 1 \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \cos\alpha = \cos\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\therefore$ So, the correct option is (B).

Ex 3.4.

Q.1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$. We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad \left[\because R_2 \to R_2 - 2R_1, \; R_2 \to \frac{1}{5}R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A \quad [\because R_1 \to R_1 + R_2]$$

$$A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \quad [\because AA^{-1} = I]$$

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Q.2 $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We know that $A = IA$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} A$

$[\because R_1 \leftrightarrow R_2]$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} A \quad [\because R_2 \to R_2 - 2R_1]$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} A \quad [\because R_2 \to (-1)R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A \quad [\because R_1 \to R_1 - R_2]$

$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

Q.3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$. We know that $A = IA$

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

$[\because R_2 \to R_2 - 2R_1]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A \quad [\because R_1 \to R_1 - 3R_2]$

$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

$[1 - 3 \times 0$
$= 1 - 0$
$= 1$

$0 - 3 \times 1$
$= 0 - 3$
$= -3$

$1 - 3 \times (-2)$
$= 1 + 6 =$

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Q. 4. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$. We know that $A = IA$

$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$

$[\because R_1 \to \frac{1}{2} R_1]$

$\begin{bmatrix} 1 & \frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{5}{2} & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 - 5R_1]$

$\Rightarrow \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 5 & -2 \end{bmatrix} A \quad [\because R_2 \to (-2) R_2]$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A \quad [\because R_1 \to R_1 - \frac{3}{2} R_2]$

$A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

Q. 5. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$, We know that $A = IA$

$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$

$[\because R_1 \to \frac{1}{2} R_1]$

$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{7}{2} & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 - 7R_1]$

$\Rightarrow \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -7 & 2 \end{bmatrix} A \quad [\because R_2 \to 2R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \quad [\because R_1 \to R_1 - \frac{1}{2} R_2]$

$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$